

Theoretical perspective on rare and radiative charm decays

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Recent experimental bounds on rare charm decays offer a chance to improve our theoretical understanding of physics present in $c \rightarrow u\gamma$ and $c \rightarrow ul^+l^-$ transitions. Standard Model and New Physics contributions are reviewed for inclusive and exclusive $D \rightarrow V\gamma$, $D^+ \rightarrow \pi^+l^+l^-$, $D \rightarrow l^+l^-$ decays. Observables important for search of New Physics are discussed. Possibility to observe CP violation in rare charm decays is questioned.

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1 Introduction

At low energies New Physics (NP) was expected to be seen indirectly in the down quark sector. The LHC offered chance for direct search of NP. Although, NP is not found yet directly at high energies, there are a number of reasons why we still expect to find its presence. For example on experimental side, in B physics tensions between SM expectations and experimental results are found. It was noticed that in $B \rightarrow K^* \mu^+ \mu^-$ observable, known as P'_5 , deviates for about 3σ [1] from the SM prediction, the ratio $R_{D^*}^{\tau, l} = BR(B \rightarrow D \tau \nu_\tau) / BR(B \rightarrow K D \mu \nu_\mu)$ exhibit 3.8σ deviations [2] and $R_K = BR(B \rightarrow K \mu^+ \mu^-)_{q^2 \in [1, 6] \text{ GeV}^2} / BR(B \rightarrow K e^+ e^-)_{q^2 \in [1, 6] \text{ GeV}^2}$ has 2.6σ discrepancy from the SM value [3]. Two of these observables in $B \rightarrow K^{(*)}$ transitions are result of flavor changing neutral current processes (FCNC), while the one for $B \rightarrow D^{(*)}$ is a result of charge current. These anomalous results stimulated numerous studies of NP in B meson system. One should keep in mind that all other B meson physics observables offer additional constraints to new physics.

Top quark physics seems to be important for NP searches in the up-like quark sector. Properties and dynamics of top quarks attract a lot of attention on experimental and theoretical side. However, there is no indication yet about NP presence. The question one can ask: is there any chance to observe NP effects in charm FCNC physics? The constraints on NP in semileptonic charm decays driven by charge currents have been discussed in Ref. [4]. On the other hand, FCNC rare charm processes are accessible in radiative or semileptonic decays in which transitions $c \rightarrow u \gamma$ and $c \rightarrow ul^+ l^-$ occur. The main obstacle to search for NP in rare charm decay is the presence of many non-charm resonances in the vicinity of D mesons masses. Strong role of GIM mechanism is very important in charm FCNC dynamics. The interplay of CKM parameters and masses of down-like quarks leads to strong suppression of all FCNC in D meson processes. In addition, long distance contributions overshadow short distance effects. The main issue is how to separate information on short distance dynamics, either within SM or in its extensions. This is a longstanding problem in rare charm decays. Three years ago flavor community was concerned about discrepancy between measured and expected CP violating asymmetry in charm decays [5]. Although this discrepancy seems to disappear, many studies and additional checks of the observed anomaly in rare charm decays were performed. The question on observability of CP violation in charm rare decays, should be answered. In Sec. 2 contributions to $c \rightarrow u \gamma$ and $c \rightarrow ul^+ l^-$ decay modes are reviewed. The exclusive weak radiative $D \rightarrow V \gamma$ decays are discussed in Sec. 3. and $D \rightarrow \mu^+ \mu^-$, $D \rightarrow P(P') \mu^+ \mu^-$ were analysed in Sec. 4. Tests of CP violation in charm meson decays with the leptons in the final state are discussed in Sec. 5. Last section contains the summary.

2 Inclusive decay modes: $c \rightarrow u\gamma$ and $c \rightarrow ul^+l^-$

The $c \rightarrow u\gamma$ and $c \rightarrow ul^+l^-$ transitions within SM can be approached by the effective low-energy Lagrangian:

$$\mathcal{L}_{eff}^{SD} = -\frac{4G_F}{\sqrt{2}} V_{cb}^* V_{ub} \sum_{i=7,9,10} C_i Q_i, \quad (1)$$

The operators are then:

$$\begin{aligned} Q_7 &= \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) c, \\ Q_9 &= \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{l} \gamma^\mu l, \\ Q_{10} &= \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{l} \gamma^\mu \gamma_5 l. \end{aligned} \quad (2)$$

In (1) C_i denote as usual Wilson coefficients (they are determined at the scale $\mu = m_c$), $F_{\mu\nu}$ is the electromagnetic field strenght and $q_L = \frac{1}{2}(1 - \gamma_5)q$. In the case of $c \rightarrow u\gamma$ decay only C_7 contributes, while in the case of $c \rightarrow ul^+l^-$ all three Wilson coefficients are present. The QCD corrections enhance the rate to $BR(c \rightarrow u\gamma)_{SM} = 2.5 \times 10^{-8}$ [6, 7]. Within Standard model the short distance contribution coming from $Q_{7,9}$ leads to the branching ratio $BR(D \rightarrow X_u e^+ e^-)_{SM}^{SD} \simeq 3.7 \times 10^{-9}$ [8, 9, 10]. Long distance contributions overshadow the short distance one with $BR(D \rightarrow X_u e^+ e^-)_{SM}^{LD} \sim \mathcal{O}(10^{-6})$ [8, 9].

3 Exclusive decay modes: $D \rightarrow V\gamma$

Previous studies of the these decays were based on the knowledge of non-leptonic weak decays of charm mesons to two light vectors and then vector meson dominance was assumed to predict rates for $D \rightarrow V\gamma$ [11], or a model of charm mesons as heavy mesons accompanied by hidden symmetry approach for the vector mesons as done in [12]. There are also QCD sum rules calculation done by authors of Ref. [13] and more recent one in Ref. [14]. It was found that the amplitudes fulfil relations $\mathcal{A}(D^0 \rightarrow \bar{K}^{*0} \gamma) \simeq \mathcal{A}(D^0 \rightarrow \rho^0 \gamma)$ and $\mathcal{A}(D_s^+ \rightarrow \rho^+ \gamma) \simeq \mathcal{A}(D^+ \rightarrow \rho^+ \gamma)$ [13] that led to the predictions for the branching ratios $BR(D^0 \rightarrow \bar{K}^{*0} \gamma) \simeq 1.5 \times 10^{-4}$, $BR(D^0 \rightarrow \rho^0 \gamma) \simeq 3.1 \times 10^{-6}$, $BR(D_s^+ \rightarrow \rho^+ \gamma) \simeq 2.8 \times 10^{-5}$ and $BR(D^+ \rightarrow \rho^+ \gamma) \simeq 2.7 \times 10^{-6}$. On the experimental side there only two results for the branching ratios $BR(D^0 \rightarrow \bar{K}^{*0} \gamma)_{exp} \simeq 3.27(34) \times 10^{-4}$ and $BR(D^0 \rightarrow \phi \gamma)_{exp} \simeq 2.70(35) \times 10^{-5}$. One might update calculation of [11] including more recent results for the $D \rightarrow V_1 V_2$ helicity amplitudes. However, the relative phases of different contributions are still not possible to obtain and only range of the values for the branching ratios can be given: $BR(D^0 \rightarrow \bar{K}^{*0} \gamma) \simeq (2.8 - 4.9) \times 10^{-4}$ and $BR(D^0 \rightarrow \phi \gamma) \simeq (2.8 - 4.1) \times 10^{-5}$.

4 $D^0 \rightarrow \mu^+\mu^-$, $D \rightarrow P\mu^+\mu^-$ and $D \rightarrow P_1P_2\mu^+\mu^-$

The LHCb collaboration improved the bound on the rate $BR(D^0 \rightarrow \mu^+\mu^-) < 6.2(7.6) \times 10^{-9}$ [15] and for the first time, they determined limits on the branching fractions in several dilepton invariant mass bins in $BR(D^+ \rightarrow \pi^+\mu^+\mu^-) < 7.3(8.3) \times 10^{-8}$ [16]. At the low dilepton invariant mass region $0.25 \text{ GeV} \leq m_{\mu\mu} \leq 0.525 \text{ GeV}$ the LHCb collaboration found upper bound on the rate $BR(D^0 \rightarrow \mu^+\mu^-)_{l.e.b} < 2 \times 10^{-8}$, while at high dilepton invariant mass $1.25 \text{ GeV} \leq m_{\mu\mu} \leq 2.0 \text{ GeV}$, $BR_{h.e.b}(D^0 \rightarrow \mu^+\mu^-) < 2.6 \times 10^{-8}$ [16], at 90% confidence level. These two results enable to constrain size of the Wilson coefficients entering effective Lagrangian (1). This puts then limits on NP contributions in $c \rightarrow ul^+l^-$ in a model independent way. For analyses of NP effects in $D^+ \rightarrow \pi^+\mu^+\mu^-$ one needs matrix elements of $\bar{u}_L\gamma_\mu c_L$ and $\bar{u}\sigma^{\mu\nu}(1+\gamma_5)c$. We follow here standard parametrisation of these matrix elements described in [17, 18]: $\langle \pi(k) | \bar{u}\gamma^\mu(1-\gamma_5)c | D(p) \rangle = (p+k)^\mu f_+(q^2) + (p-k)^\mu f_-(q^2)$. For the $f_+(q^2)$ form factor we use Bećirević-Kaidalov parametrisation [19] as given in detail in [18]. The tensor current matrix element is parametrised as $\langle \pi(k) | \bar{u}\sigma^{\mu\nu}c | D(p) \rangle = i2f_T(q^2)/m_D + m_\pi[(p+k)^\mu q^\nu - (p+k)^\nu q^\mu]$. The HFAG report [20] was used to for the relevant parameters present in $f_+(q^2)$ and $f_T(q^2)$ as given in [18]. Based on the effective Lagrangian (1), the most general expression for the short distance amplitude can be written as:

$$\begin{aligned} \mathcal{M}_{SD}(D^+(p) \rightarrow \pi^+(k)\mu^+(p_+)\mu^-(p_-)) &= \frac{G_F}{\sqrt{2}}\lambda_b\alpha\left\{\left[\frac{m_c}{m_D+m_\pi}\frac{4}{\pi}C_7f_T(q^2)\right.\right. \\ &\left.\left.+\frac{1}{\pi}C_9f_+(q^2)\right]\bar{u}(p_-)p_\alpha\gamma^\alpha v(p_+) + \frac{1}{\pi}C_{10}f_+(q^2)\bar{u}(p_-)p_\alpha\gamma^\alpha\gamma_5 v(p_+)\right\}. \end{aligned} \quad (3)$$

The branching ratio for $D \rightarrow \mu^+\mu^-$ can be written as:

$$BR(D^0 \rightarrow \mu^+\mu^-) = \frac{1}{\Gamma_D} = \frac{G_F^2\alpha^2}{64\pi^3}|V_{cb}^*V_{ub}|^2f_D^2m_D^3\sqrt{1-\frac{4m_\mu^2}{m_D^2}}\frac{2m_\mu^2}{m_D^2}|C_{10}|^2. \quad (4)$$

Within SM long distance dynamics can be described by the processes $D^+ \rightarrow \pi^+V^0$ with $V^0 \rightarrow \rho^0, \omega$ and ϕ in which then V^0 decays to $\mu^+\mu^-$ pair, presented in details in Ref. [17] for the contribution of $D^+ \rightarrow \pi^+\rho^0(\omega)$ and updated for the $D^+ \rightarrow \pi^+\phi \rightarrow \pi^+\mu^+\mu^-$ in Ref. [18]. The existing experimental upper bound in the non-resonance regions indicates that the long distance contribution is fairly suppressed. On Fig. 1 we present SM contributions to the differential branching fraction for $D^+ \rightarrow \pi^+\mu^+\mu^-$ as a function of dilepton invariant mass. We also give experimental upper bound for the differential branching ratio as found by LHCb [16].

If one considers contributions of NP and if new particle is a new scalar or pseudoscalar particle mediating the decay $c \rightarrow ul^+l^-$, then the same particle would contribute to $D^0 - \bar{D}^0$ oscillations and the physical observable from this process restricts

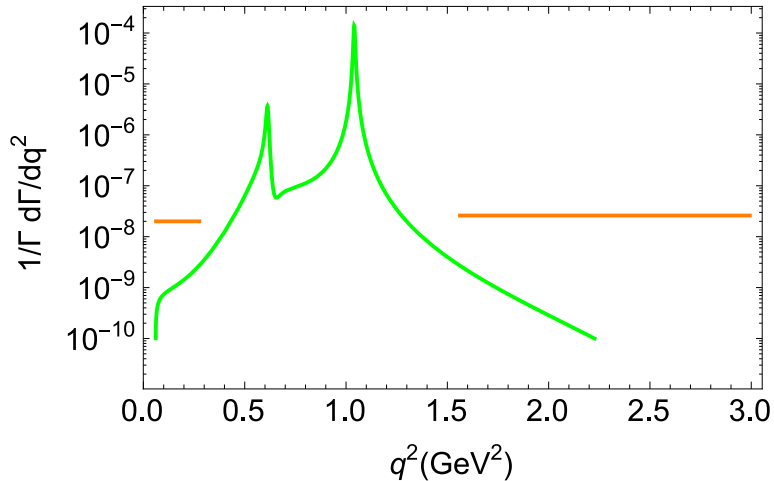


Figure 1: Long distance contributions to differential branching ratio for $D^+ \rightarrow \pi^+ \mu^+ \mu^-$, as a function of dilepton invariant mass (green) and the LHCb bounds for the non-resonant bins (orange).

the couplings of this operator. The same holds for the flavor changing Z or new Z' boson. In the case that NP is generated at the loop level in $c \rightarrow u \ell^+ \ell^-$ then it contributes to $D^0 - \bar{D}^0$ at the loop level too, as presented in Fig. 2. In addition to differential branching ratios at low/high dilepton invariant mass NP detection in these decays was also discussed by suggesting new observables. It was found that two angular asymmetries, namely the T-odd di-plane asymmetry and the forward-backward dilepton asymmetry offer direct tests of new physics due to tiny SM backgrounds [10].

It is important to mention that outside resonance regions of $D^+ \rightarrow \pi^+ \mu^+ \mu^-$ the long distance, as well as SM short distance contributions are more than two orders of magnitude smaller than the total branching ratio. Experimental results for the differential decay width distribution at the low/high dilepton invariant mass bins can be explained by the contributions of the effective Wilson coefficients. We allow only one Wilson coefficient at the time to have maximal value. At the same time $D^0 \rightarrow \mu^- \mu^-$ can give bound on the C_{10} . Results are presented in Table 1. It turned out that the upper bound on $D^0 \rightarrow \mu^+ \mu^-$ is more restrictive on C_{10} Wilson coefficient than any of the differential branching ratios for $D^+ \rightarrow \pi^+ \mu^+ \mu^-$ in dilepton invariant mass bins. In Table 1 we use notation $\tilde{C}_i = V_{ub} V_{cb}^* C_i$. The differential rate at high dilepton invariant mass bin is more restrictive for the effective Wilson coefficients than the differential rate at low dilepton invariant mass bin. Many models of NP were discussed in literature as supersymmetry with or without R-parity violation, new vector-like quarks, leptoquarks [17, 18], Little Higgs models [10]. All of these models modify some of Wilson coefficients C_i , $i = 7, 9, 10$, however, they are still smaller than the bounds given in Table 1. The detailed analysis of the semileptonic

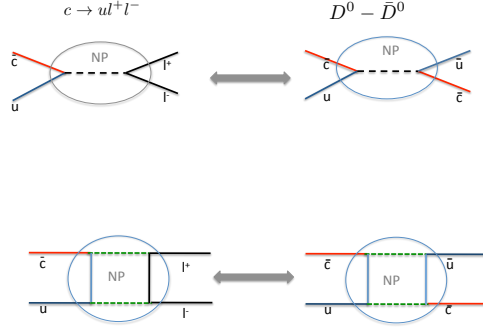


Figure 2: NP contributions to $c \rightarrow ul^+l^-$ and $D^0 - \bar{D}^0$ at tree and loop level.

four body $D \rightarrow hhl^+l^-$ decays was done in the work of Ref. [21]. The dominant long distance contributions (bremsstrahlung and hadronic effects) are calculated and total branching ratios and the Dalitz plots are presented. Assuming vector meson dominance, it was found $BR(D^0 \rightarrow K^-\pi^+l^+l^-) \sim 10^{-5}$, $BR(D^0 \rightarrow \pi^-\pi^+l^+l^-) \sim 10^{-6}$, $BR(D^0 \rightarrow K^-K^+l^+l^-) \sim 10^{-7}$ and $BR(D^0 \rightarrow K^+\pi^-l^+l^-) \sim 10^{-8}$.

5 Direct CP violation in rare D decays

In 2011 the LHCb collaboration [5] found rather large CP violation in the difference of CP violating asymmetries for $D \rightarrow \pi^+\pi^-(K^+K^-)$. Using the updated result of the LHCb collaboration [22], HFAG [20] produced the world average CP asymmetry, $\Delta A_{CP} = A_{CP}(D \rightarrow K^+K^-) - A_{CP}(D \rightarrow \pi^+\pi^-) = (-0.253 \pm 0.104)\%$. Whether CP violating asymmetry is present in $D \rightarrow \pi^+\pi^-(K^+K^-)$ or not is still an open issue. For charm meson decays, the CP violating asymmetry is defined as:

$$A_{CP} = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}. \quad (5)$$

The authors of Ref. [23] investigated CP violating observables in $D \rightarrow V\gamma \rightarrow P^+P^-\gamma$ decays. If CP violation is due to NP effects [23], then it is most likely a result of the chromomagnetic operator Q_8 contribution [24]. In the case of rare D decays CP violation results from mixing of Q_8 into Q_7 under QCD renormalization. $\text{Im}[C_7^{NP}(m_c)] \simeq \text{Im}[C_8^{NP}(m_c)] \simeq 0.02 \times 10^{-2}$. NP contribution can be comparable in size with the real part of the SM $|C_7^{SM-eff}(m_c)| = (0.5 \pm 0.1) \times 10^{-2}$. This implies

Max. \tilde{C}_i	$(D \rightarrow \pi\mu\mu)_{l.i.b.}$	$(D \rightarrow \pi\mu\mu)_{h.i.b.}$	$D \rightarrow \mu\mu$
$ \tilde{C}_7 $	0.54	0.46	-
$ \tilde{C}_9 $	1.33	0.91	-
$ \tilde{C}_{10} $	1.32	0.68	0.63
$ \tilde{C}_9 = - \tilde{C}_{10} $	0.91	0.54	-

Table 1: Maximally allowed value of the Wilson coefficients, $\tilde{C}_i = V_{ub}V_{cb}^*C_i$, calculated in the non-resonance regions of $D^+ \rightarrow \pi^+\mu^+\mu^-$ at the low lepton invariant mass $m_{\mu\mu} \in [0.25, 0.525]$ GeV, denoted by *l.i.b.* and at the high invariant mass region $m_{\mu\mu} \in [1.25, 2.0]$ GeV, denoted by *h.i.b.*, and from the upper bound on the rate of $\text{Br}(D^0 \rightarrow \mu^+\mu^-) < 7.6 \times 10^{-9}$. The last row gives the maximal value for the case $|\tilde{C}_9| = -|\tilde{C}_{10}|$.

that if the phase of long distance contribution can be neglected and the relative strong phase is maximal, the CP asymmetry can reach the $\mathcal{O}(1\%)$ level. The current world average of the CP violating asymmetry in ΔA_{CP} , following the work of [23] leads to a CP asymmetry in $D \rightarrow K^+K^-\gamma$ of the order 1%.

We found that in $D \rightarrow P\ell^+\ell^-$ there is a possibility to study CP violation observables [18]. New CP violating effects in rare decays $D \rightarrow P\ell^+\ell^-$ are consequence of the interference of resonant part of the long distance contribution and the new physics affected short distance contribution. The observables, the differential direct CP asymmetry and partial decay width CP asymmetry can be introduced in a model independent way. Among all decay modes the simplest one for the experimental searches are $D^+ \rightarrow \pi^+\ell^+\ell^-$ and $D_s^+ \rightarrow K^+\ell^+\ell^-$. Only when third generation is included there is a possibility to obtain non-vanishing imaginary part: $\text{Im}(\lambda_b/\lambda_d) = -\text{Im}(\lambda_s/\lambda_d)$. The CP violating parts of the amplitude are suppressed by a very small factor $\lambda_b/\lambda_d \sim 10^{-3}$ with respect to the CP conserving ones and therefore the CP violating effects should be very small. In vicinity of the ϕ resonant peak, the long distance amplitude for $D^+ \rightarrow \pi^+\mu^+\mu^-$ decay is well approximated by non-factorizable contributions of four-quark operators in \mathcal{H}^s . The width of ϕ resonance is very narrow ($\Gamma_\phi/m_\phi \approx 4 \times 10^{-3}$) and well separated from other vector resonances in the q^2 spectrum of $D \rightarrow P\ell^+\ell^-$. Relying on vector meson dominance hypothesis the q^2 -dependence of the decay spectrum close to the resonant peak follows the Breit-Wigner shape [8, 17]. With $\mathcal{A}_{LD}^\phi = \overline{\mathcal{A}}_{LD}^\phi$ the differential direct CP violation becomes

$$a_{CP}(\sqrt{q^2}) \equiv \frac{|\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2} \sim \text{Im} \left[\frac{\lambda_b}{\lambda_s} C_7 \right]. \quad (6)$$

The asymmetry can reach $a_{CP} \sim 1\%$ (see discussion in [18]). In addition, a CP

Decay mode	size	Reference
$D \rightarrow \rho(\omega)\gamma$	$\leq 3\%$	[14]
$D \rightarrow K^+K^-\gamma$	$\leq 1\%$	[23]
$D \rightarrow X_u l^+ l^-$	$\leq 3\%$	[?]
$D^+ \rightarrow \pi^+ \mu^+ \mu^-$	$\leq 2\%$	[18]
$D^+ \rightarrow hh \mu^+ \mu^-$	$\leq 1\%$	[21]

Table 2: CP violating asymmetries for charm rare decays, the size and the original reference.

asymmetry of a partial width in the range $m_1 < m_{\ell\ell} < m_2$ can be defined as:

$$A_{\text{CP}}(m_1, m_2) = \frac{\Gamma(m_1 < m_{\ell\ell} < m_2) - \bar{\Gamma}(m_1 < m_{\ell\ell} < m_2)}{\Gamma(m_1 < m_{\ell\ell} < m_2) + \bar{\Gamma}(m_1 < m_{\ell\ell} < m_2)}, \quad (7)$$

where Γ and $\bar{\Gamma}$ denote partial decay widths of D^+ and D^- decays, respectively, to $\pi^\pm \mu^+ \mu^-$. A_{CP} can be related to the differential asymmetry $a_{\text{CP}}(\sqrt{q^2})$ as described in [18]. The largest possible asymmetries are of the order few percent. The new physics detection in these decay modes was also discussed. It was found that two angular asymmetries, namely the T-odd diplane asymmetry and the forward-backward dilepton asymmetry offer direct tests of New Physics due to tiny Standard model backgrounds. If supersymmetric and Z' -enhanced scenarios are assumed, and if the size of Wilson coefficients C_9 and C_{10} is compatible with the observed CP asymmetry in nonleptonic charm decays and flavor constraints, it was found in [21] that new physics effects in $D^0 \rightarrow h_1 h_2 l^+ l^-$ might reach the percent level.

If supersymmetric and Z' -enhanced scenarios are assumed and if the size of Wilson coefficients C_9 and C_{10} is compatible with the observed CP asymmetry in nonleptonic charm decays and flavor constraints, it was found in [21] that new physics effects in $D^0 \rightarrow h_1 h_2 l^+ l^-$ might reach the $\sim 1\%$ level.

6 Summary

Within SM rare charm decays are fully dominated by long distance dynamics. Recent results of LHCb experiment on $D \rightarrow \mu^+ \mu^-$ and $D^+ \rightarrow \pi^+ \mu^+ \mu^-$ enable to determine bounds on effective Wilson coefficients: C_i , $i = 7, 9, 10$.

It was found that some signals of new physics might arise in $D \rightarrow K^+ K^- \gamma$, as well as in decays with the leptonic pair in the final state $D \rightarrow X_u l^+ l^-$, $D^+ \rightarrow \pi^+ \mu^+ \mu^-$, $D^+ \rightarrow hh \mu^+ \mu^-$. In discussion of NP in rare charm decays it is necessary to check whether $D^0 - \bar{D}^0$ oscillations give additional constraints on the couplings of new physics. At the same time, K and B physics might for doublets of up-like quarks interacting with NP particle, give very restrictive bounds.

The possible presence of CP violation induced by new physics in charm nonleptonic decays has stimulated a number of studies. The three body $D \rightarrow P\ell^+\ell^-$ decay decays is particularly interesting, since one can focus on the CP asymmetry around the ϕ resonant peak in spectrum of dilepton invariant mass. The appropriate observables, the differential direct CP asymmetry and the partial decay width CP asymmetry can be introduced in a model independent way.

Although long distance dynamics overshadows short distance contributions in rare charm decays, more precise measurements and improved knowledge of hadronic quantities, might uncover presence of New Physics.

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